6 Permutations and Combinations

People with diabetes might need to take supplementary insulin to help keep their blood sugar levels within a normal range. Insulin is a kind of protein that is present in humans. It's job is to keep the quantity of sugar in the body in check so that it doesn't become too high or too low. Insulin is made up of 51 amino acids that are organized in a certain order or permutation. Any rearrangement that differs from this usual sequence renders the protein defective, resulting in illnesses like diabetes. The body has a system in place to guarantee that this sequence is followed and that the proper protein is produced.

Topic Notes

- *Fundamentals of Counting and Permutations*
- *Combinations and its Applications*

FUNDAMENTAL OF COUNTING AND PERMUTATIONS 1

TOPIC 1

FUNDAMENTAL PRINCIPLE OF COUNTING

The foundation of permutations and combinations are two essential ideas of counting.

Fundamental Principle of Multiplication (FPM)

The fundamental principle of multiplication is also known as multiplication principle. If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is $m \times n$.

If two events occur in a and b ways respectively, then the total number of occurrence of the events are given by $a \times b$.

e.g. In a school, there are 200 boys and 150 girls. The teacher wants to select a boy and a girl to represent the school in inter school competition.

Here, the teacher can select a boy in 200 ways and a girl in 150 ways. So, by principle of multiplication, the teacher can select a boy and a girl in $200 \times 150 = 30000$ ways.

Important

- *The above principle can be generalised for any finite number of events. For example, for 3 events, the principle is as follow:*
- *'If an event can occur in a different ways, following which another event can occur in b different ways, following which a third event can occur in c different ways, then the total number of occurrence to 'the events in the given order is $a \times b \times c$'.*

Fundamental Principle of Addition (FPA)

Let two events are performing independently in a and b ways respectively, then either of the two events can occur in $(a + b)$ ways.

Example 1.1: How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than once? [NCERT]

Ans. The telephone numbers are of 5 digits and first two numbers are 6 and 7.

Hence, numbers are of the form

6	7	___	___	___
↓	↓	↓	↓	↓
1^{st} digit	2^{nd} digit	3^{rd} digit	4^{th} digit	5^{th} digit

Position	Number of different ways
1^{st} (fix)	1
2^{nd} (fix)	1
3^{rd}	8
4^{th}	7
5^{th}	6

Digits between 0 to 9 = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$\therefore$ Total number of digits between 0 to 9 = 10

Now,

1^{st} and 2^{nd} place is fix.

3^{rd} place: After fixing 67, there are $(10 - 2) = 8$ ways fill 3^{rd} place.

4^{th} place: After fixing 67 and 3^{rd} place there are only $10 - 3 = 7$ ways to fill 4^{th} place.

5^{th} place: After fixing 67, 3^{rd} and 4^{th} place there are $10 - 4 = 6$ ways to fill 5^{th} place.

Thus, the total number of telephone numbers that can be formed

$= 1 \times 1 \times 8 \times 7 \times 6 = 336$

Example 1.2: Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other? [NCERT]

Ans. A signal can have only 2 flags

→ First flag

→ Second flag

Flag Position	Number of different ways
First	5
Second	4

Total no. of signals generated

$= 5 \times 4 = 20$

TOPIC 2

PERMUTATIONS

A permutation is an arrangement of a number of objects in a definite order, taken some or all at a time.

The number of permutations of n different objects, taken r at a time where, $0 \le r \le n$ is given by

$$^nP_r = \frac{n!}{(n-r)!}$$

Here, ! is termed as factorial.

Factorial Notations (!)

In earlier classes, we came across the products of numbers in the form 1 × 2, 2 × 1 etc. So to make this more convenient, we use a special notation, i.e.,

$1! = 1$

$2! = 1 \times 2$

$3! = 3 \times 2 \times 1$

$4! = 4 \times 3 \times 2 \times 1 = 4 \times 3! = 4 \times 3 \times 2!$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 5 \times 4! = 5 \times 4 \times 3!$

and so on.

Important

➥ *The notation n! represent the product of first n natural numbers.*

Zero Factorial

It does not make any sense to define 0! as product of the integers from 1 to 0. So, we define 0! = 1.

Caution

➥ *Students should know that factorial of fractions or negative integers are not defined. n! is defined only for whole numbers, i.e., for non-negative integers.*

Example 1.3: Evaluate:

(A) 8!

(B) 4! – 3!

(C) $\frac{8!}{6!\,2!}$

Ans. (A) $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

(B) $4! - 3! = (4 \times 3 \times 2 \times 1) - (3 \times 2 \times 1)$

$= 24 - 6$

$= 18$

(C) $\frac{8!}{6!\,2!} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1} = 28$

Example 1.4: If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, find x.

Ans. We have,

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

$$\Rightarrow \quad \frac{8!}{6!} + \frac{8!}{7!} = x$$

$$\Rightarrow \quad \frac{8 \times 7 \times 6!}{6!} + \frac{8 \times 7!}{7!} = x$$

$$\Rightarrow \quad 56 + 8 = x$$

$$\Rightarrow \quad x = 64$$

Permutations when all the Objects are Distinct

Theorem 1

The number of permutations of n different objects taken r at a time, where $0 < r \le n$ and the objects do not repeat, is $n(n-1)(n-2) \ldots (n-r+1)$, which is denoted by ^nP_r or $P(n, r)$.

i.e., $P(n, r) = {}^nP_r = \frac{n!}{(n-r)!}, 0 \le r \le n$

Proof: We know that

$$P(n, r) = {}^nP_r = n(n-1)(n-2)(n-3)\ldots[n-(r-1)]$$

On multiplying numerator and denominator by

$(n-r)(n-r-1)\ldots 3 \times 2 \times 1$, we get

$$^nP_r = \left[\frac{n(n-1)(n-2)\ldots(n-(r-1)) \times (n-r)(n-r-1)\ldots 3.2.1}{(n-r)(n-r-1)\ldots 3.2.1}\right]$$

$$= \frac{n(n-1)(n-2)\ldots 3.2.1}{(n-r)(n-r-1)\ldots 3.2.1}$$

$$= \frac{n!}{(n-r)!}, 0 \le r \le n$$

In particular,

(i) When $r = 0$, then $^nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$

(ii) When $r = n$, then $^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$ $[\because 0! = 1]$

Important

➥ *Number of permutations of n different things, taken all at a time = n!.*

Permutations with Repetitions

When repetition of objects is allowed, then the number of permutations can be obtained with the help of the following theorem.

Theorem 2

The number of permutations of n different objects taken r at a time, when each object may be repeated any number of times in each arrangement, is n^r (permutation with repetitions).

Important

➥ The number of permutations of n different objects taken all at a time, when each may be repeated any number of times in each arrangement, is n^n.

Permutations when all the Objects are not Distinct Objects

When all the items are not distinct, i.e. some of them are of the same kind, we can use the following theorems to calculate the number of permutations.

Theorem 3

The number of permutations of n objects, where p objects are of the same kind and rest are all different $= \frac{n!}{p!}$.

In fact, we have a more general theorem.

Theorem 4

The number of permutations of n objects, where P_1 objects are of one kind, P_2 are of second kind, ..., P_k are of k^{th} kind and the rest, if any, are of different kind is,

$$\frac{n!}{P_1!P_2!\ldots P_k!}$$

Example 1.5: Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4,5 if no digit is repeated. How many of these will be even? **[NCERT]**

Ans. Let the 4 digit number be

-- -- -- --

Total number of digits (1, 2, 3, 4, 5) = 5

So, $n = 5$

We need to make 4-digit number, so we take 4 digits at a time,

So, $r = 4$

Number of 4-digit numbers $= {}^nP_r$

$$= {}^5P_4$$

$$= \frac{5!}{(5-4)!}$$

$$= \frac{5!}{1!}$$

$= 5!$ $\quad [\because 1! = 1]$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

Thus, number of 4-digit numbers = 120

Now, we have to find even 4-digit numbers

-- -- -- --

In an even number, unit's place should be even

In our digits (1, 2, 3, 4, 5), only 2, 4 are even.

Hence, either 2 or 4 can be at the unit place

So, we fix 2 at unit place and then find the total ways.

Numbers where digit at unit place = 2

-- -- -- 2

Remaining 3 places are to be filled with digits (1, 3, 4, 5)

Hence,

n = number of digits left = 4

and r = number of digits to be taken = 3

Number of ways in which remaining 3 places are filled

$$= {}^4P_3$$

$$= \frac{4!}{(4-3)!}$$

$$= \frac{4!}{1!}$$

$= 4!$ $\quad [\because 1! = 1]$

$= 4 \times 3 \times 2 \times 1$

$= 24$

Similarly, if we fix 4 at unit place as then

Number of ways = 24

Total number of 4 digit even numbers

$= 24 + 24 = 48$

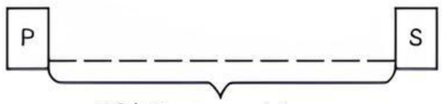
Example 1.6: In how many ways can the letters of the word PERMUTATIONS be arranged if the

(A) words start with *P* and end with *S*?

(B) vowels are all together?

(C) there are always 4 letters between *P* and *S*? **[NCERT]**

Ans. (A) Let first position be P and last position be S (both are fixed)



10 letters remaining

Since, letters are repeating

Hence, we use the formula $\frac{n!}{p_1!p_2!p_3!}$

Total number of remaining letters $= n = 10$

$[\therefore P$ and S are fixed$]$

and since there are 2*T*'s

Therefore, $p_1 = 2$

Hence, total arrangements $= \frac{10!}{2!}$

$= 1814400$

(B) Vowels in word PERMUTATION = (EUAIO)

We write EUAIO as a single object.

So, our letters become EAUIOPRMTTNCS

Let's arrange them now

Arranging 5 vowels	Arranging remaining letters
Since, vowels are coming together, they can be EAUIO, OAEUI, AEIOU and so on. Total letters in AEIOU = 5 Total permutations of 5 letters $= {}^5P_5$ $= \frac{5!}{(5-5)!}$ $= \frac{5!}{0!} = \frac{5!}{1}$ = 120	Letters we need to arrange $= 7 + 1 = 8$ Here are 2*T*. so, letters are repeating. We use the formula $= \frac{n!}{p_1!p_2!p_3!}$ Total letters = $n = 8$ As, there are 2T's. $\therefore p_1 = 2$ Total arrangements $= \frac{8!}{2!}$

Hence,
Total number of arrangements

$$= \frac{8!}{2!} \times 120 = 2419200$$

(C)

Position of *P*	Position of *S*
1^{st}	6^{th}
2^{nd}	7^{th}
3^{rd}	8^{th}
4^{th}	9^{th}
5^{th}	10^{th}
6^{th}	11^{th}
7^{th}	12^{th}
8^{th}	13^{th} (Not possible as there are only 12 letters)

So, there are total 7 cases where there are 4 letters between *P* and *S* but when *P* is before *S*.

There can be a case where S is before *P*.

Thus, total cases = 7 × 2 = 14 *cases*

Now, let's find permutations of letters in 1^{st} case

[P] _ _ _ _ [S] _ _ _ _ _ _

Case 1^{st}

Since Position of *P* and *S* are fixed, we need to arrange remaining letters *i.e.*, (*E, R, M, U, T, A, T, I, O, N*).

Since, T is repeating, we use this formula

$$= \frac{n!}{p_1!p_2!p_3!}$$

Number of remaining letters = 10

$n = 10$

There are 2*T*'s

$p_1 = 2$

Number of arrangements = $\frac{10!}{2!}$

Thus, Total number of arrangements

$$= 14 \times \frac{10!}{2!}$$

$$= 14 \times \frac{10!}{(2 \times 1)}$$

$$= 7 \times 10!$$

$$= 7 \times 3{,}628{,}700$$

$$= 25{,}401{,}600$$

Important

- *If r particular things out of n different things are to be together, then we count these r particular things as one thing and remaining (n – r) things as separate things.*
Then, total number of things = (n – r) + 1
= n – r + 1.
But, r particular things can also be arranged among themselves in r! ways.
∴ Required number of permutations = (n – r + 1)! r!
If r particular things are identical, then the required number of permutations = (n – r + 1)!
- *The number of permutations of n objects taken r at a time, when a particular object is taken in each arrangement, is r. ${}^{n-1}P_{r-1}$.*
- *The number of permutations of n objects taken r at a time, when a particular object is never taken in each arrangement, is ${}^{n-1}P_r$.*
- *The number of permutations of n different objects taken r at a time in which two specific objects always occur together, is 2!(r – 1). ${}^{n-2}P_{r-2}$.*

Example 1.7: If the different permutations of all the letters of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E? [NCERT]

Ans. In dictionary, words are given alphabetically, we need to find words starting before E i.e., starting with *A, B, C* or *D*.

In EXAMINATION, there is no *B, C* or *D*, hence words should start with *A*.

Words in the list before the word starting with *E*

= words starting with letter *A*

Words starting with letter *A*

A _ _ _ _ _ _ _ _ _ _

10 letters remaining

Since, letters are repeating,

Hence, we use this formula = $\frac{n!}{p_1!p_2!p_3!}$

Remaining letters = $n = 10$

Since, there are 2*I*'s, 2*N*'s in the remaining letters.

Therefore, $p_1 = 2, p_2 = 2$

Number of words formed by these letters

$$= \frac{n!}{p_1!p_2!}$$

$$= \frac{10!}{2!2!}$$

$$= \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2\times2}$$

$$= \frac{3628800}{4}$$

$$= 907200$$

Thus,

Words in the list before the word starting with *E*

= words starting with letter *A*

= 907200

Example 1.8: Case Based:

Five students Ajay, Shyam, Yojana, Rahul and Akansha are sitting in a play ground in a line.

Based on the above information answer the following questions.

(A) Assertion (A): The total number of ways of sitting arrangements of five students is 120.

Reason (R): A permutation is an arrangement of a number of objects in a definite order, taken some or all at a time.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

(B) Total number of ways of sitting arrangements of sitting if Ajay and Yojana sit together is:

(a) 60 (b) 48

(c) 72 (d) 120

(C) Total number of arrangements if Yojana and Rahul sit at extreme position, is:

(a) 24 (b) 36

(c) 48 (d) 12

(D) Find the total number of arrangements if Shyam is sitting in the middle.

(E) Find the total number of sitting arrangements, if Yojana and Rahul don't sit together.

Ans. (A) *(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).*

Explanation: There are five students.

∴ Total number of arrangement is 5! = 120

We know that, permutation is an arrangement of a number of objects in a definite order, taken some or all at a time.

(B) *(b) 48*

Explanation: Ajay and Yojana sit together.

∴ Total number of arragement = $4! \times 2! = 24 \times 2 = 48$

(C) *(d) 12*

Explanation: Total number of arrangements if Yojana and Rahul sit at in extreme position, is $2! \times 3! = 2 \times 6 = 12$

(D) Number of arrangements, if Shyam is sitting in middle, is

$$4! = 24$$

(E) Number of arrangements, if Yojana and Rahul don't sit together is

= Total number of sitting arrangements of five students – Total number of sitting arrangements when Yojana and Rahul sit together

$= 5! - (4! \times 2!)$

$= 120 - 48$

$= 72$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. The number of six-digit numbers whose all digits are odd is:

(a) 6^5 (b) 5^6

(c) $\frac{6!}{2!}$ (d) None

Ans. *(b)* 5^6

Explanation: Since, six-digit numbers whose all digits are odd are to be formed, suggests that repetition of digits is a must as available digits are 1, 3, 5, 7 and 9.

Therefore, number of 6 digit numbers = 5^6.

2. The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time is:

(a) 432 (b) 108
(c) 36 (d) 18

[NCERT Exemplar]

Ans. *(b) 108*

Explanation: ∵ The sum of digits in unit place when 3 is at their unit place = $3! \times 3$

Similarly, when unit place is 4 = $3! \times 4$

when unit place is 5 = $3! \times 5$

when unit place is 6 = $3! \times 6$

The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time, is

$$= 3!(3 + 4 + 5 + 6)$$
$$= 3 \times 2 \times 18$$
$$= 108$$

3. Without repetition, using digits 2, 3, 4, 5, 6, 8 and 0, how many numbers can be formed which lie between 500 and 1000?

(a) 90 (b) 147
(c) 60 (d) 70 [Diksha]

Ans. *(a) 90*

Explanation: There are total 7 digits given –2, 3, 4, 5, 6, 8 and 0.

Between 500 and 1000 i.e., from 501 to 999.

All of them are 3 digit numbers.

First digit	Second digit	Third digit
5, 6 or 8 *i.e.*, 3 possibilities (1 digit gets used here)	Any digit from 7 – 1 = 6 remaining digits i.e. 6 possibilities	Any digit from 6 – 1 = 5 remaining digits i.e. 5 possibilities

∴ Total possible numbers = $3 \times 6 \times 5 = 90$

4. A boy has 9 trousers and 12 shirts. In how many different ways can he select a trouser and a shirt?

(a) 101 (b) 108
(c) 105 (d) 106 [Diksha]

Ans. *(b) 108*

Explanation: The boy can select one trouser in 9 ways.

The boy can select one shirt in 12 ways.

∴ The number of ways in which he can select one trouser and one shirt is

$$= 9 \times 12 \text{ ways}$$
$$= 108 \text{ ways}$$

5. A five-digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways, this can be done is:

(a) 216 (b) 600
(c) 240 (d) 3125

[NCERT Exemplar]

[Hint: 5 digit numbers can be formed using digits 0, 1, 2, 4, 5 or by using digits 1, 2, 3, 4, 5 since the sum of digits in these cases is divisible by 3].

Ans. *(a) 216*

Explanation: 5-digit numbers that can be formed using digits 0, 1, 2, 4, 5

4 ways	4 ways	3 ways	2 ways	1 way

No. of ways is which each place can be filled.

$$= 4 \times 4 \times 3 \times 2 \times 1 = 96$$

5-digit numbers can that be formed using digits 1, 2, 3, 4, 5 = 5!

Total number of ways = 5! + 96

$$= 120 + 96$$
$$= 216$$

6. The value of n such that $\frac{^nP_4}{^{n-1}P_4} = \frac{5}{3}$, $n > 4$ is:

(a) 11 (b) 15
(c) 12 (d) 10

Ans. *(d) 10*

Explanation: Given:

$$\frac{^nP_4}{^{n-1}P_4} = \frac{5}{3}$$

$$\Rightarrow \frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-1-4)!}} = \frac{5}{3} \quad \left[\because {^nP_r} = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \frac{\frac{n(n-1)!}{(n-4)(n-5)!}}{\frac{(n-1)!}{(n-5)!}} = \frac{5}{3}$$

$$\Rightarrow \frac{n}{(n-4)} = \frac{5}{3}$$

$$\Rightarrow 3n = 5(n-4)$$
$$\Rightarrow 3n = 5n - 20$$
$$\Rightarrow 5n - 3n = 20$$
$$\Rightarrow 2n = 20$$
$$\Rightarrow n = 10$$

7. How many three letter words can be formed using the letters of the word "TIME"?

(a) 20 (b) 24
(c) 26 (d) 12 [Diksha]

Ans. *(b) 24*

Explanation: The number of letters in the given word is 4.

The number of 3 letter words that can be formed using these four letters is $^4P_3$

$$= \frac{4!}{1!}$$

$$= 4 \times 3 \times 2$$

$$= 24$$

8. If $5\,^4P_r = {^5P_r}$, then r is:

(a) 2 (b) 4

(c) 3 (d) none of the above

Ans. *(d) none of the above*

Explanation: We have $5\,^4P_r = {^5P_{r-1}}$

$$5 \times \frac{4!}{(4-r)!} = \frac{5!}{(5-r+1)!}$$

$$(6-r)(5-r) = 0$$

$$r = 6 \text{ or } 5$$

9. The number of words which can be formed from the letters of the word MAXIMUM, if two consonants can't occur together is:

(a) 4! (b) 3! × 4!

(c) 7! (d) none of these

[Delhi Gov. QB 2022]

Ans. *(a) 4!*

Explanation: Given word MAXIMUM has 4 consonants and 3 vowels.

If two consonants cannot be kept together then they must be arranged in this way

_A_I_U_

The arrangement of vowels can be made in 3! ways (permutations of A, I, U)

The 4 consonants (3M, 1X) can be arranged in the 4 blanks in $\frac{4!}{3!}$ ways.

Hence, the total number of ways to arrange the letters $= 3! \times \frac{4!}{3!} = 4!$

Hence, there are 4! ways to arrange the letters of the word MAXIMUM such that no two consonants are together.

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

10. **Assertion (A):** If $5.^4P_r = 6.^5P_{r-1}$, then $r = 3$.

Reason (R): If $^5P_x = {^6P_{r-1}}$, then $r = 9$.

Ans. *(c) (A) is true but (R) is false.*

Explanation: We have $5.^4P_r = 6.^5P_{r-1}$

$$\Rightarrow \quad 5 \times \frac{4!}{(4-r)!} = 6 \times \frac{5!}{(5-r+1)!}$$

$$\Rightarrow \quad \frac{5!}{(4-r)!} = \frac{6 \times 5!}{(6-r)(5-r)(4-r)!}$$

$$\Rightarrow \quad (7-r)(6-r) = 6$$

$$\Rightarrow \quad 42 - r^2 - 13r = 6$$

$$\Rightarrow \quad (6-r)(5-r) = 6$$

$$\Rightarrow \quad r^2 - 11r + 24 = 0$$

$$\Rightarrow \quad (r-8)(r-3) = 0$$

$$\Rightarrow \quad r = 8, 3$$

But $r \neq 8$ as $r \leq 4$

$$\therefore \quad r = 3$$

We have, $^5P_r = {^6P_{r-1}}$

$$\Rightarrow \quad \frac{5!}{(5-r)!} = \frac{6!}{(6-r+1)!}$$

$$\Rightarrow \quad \frac{1}{(5-r)!} = \frac{6}{(7-r)(6-r)(5-r)!}$$

$$\Rightarrow \quad r^2 - 13r + 36 = 0$$

$$\Rightarrow \quad (r-4)(r-9) = 0$$

$$\Rightarrow \quad r = 4, 9$$

$$\Rightarrow \quad r = 4 \qquad [\because r \neq 9]$$

11. **Assertion (A):** The number of permutations of letters of word 'ROOT' are 10.

Reason (R): The number of permutations of letters of word 'INSTITUTE' is $\frac{9!}{2!3!}$.

Ans. *(d) (A) is false but (R) is true.*

Explanation: There are 4 letters in 'Root' of which there are 2O's and rest are different.

Therefore, the required number of arrangements

$$= \frac{4!}{2!} = 12$$

There are 9 letters in 'INSTITUTE' of which there are 2 I's, 3T's and rest are different.

Therefore, the required number of arrangements

$$= \frac{9!}{2!3!}$$

12. **Assertion (A) :** $^nP_r = \frac{n!}{(n-r)!}, 0 \leq r \leq n.$

Reason (R): $^nP_r = n(n-1)(n-2)....(n-r+1), 0 \leq r \leq n.$

Ans. (a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: We know,

$$^nP_r = n(n-1)(n-2)...(n-r+1)$$

$$= \frac{n(n-1)(n-2)...(n-r+1)(n-r)...3\times2\times1}{(n-r)(n-r-1)...3\times2\times1}$$

$$= \frac{n!}{(n-r)!}$$

13. **Assertion (A):** The value of $^6P_4$ is 360.

Reason (R): $^nP_r = P(n, r) = \frac{n!}{(n-r)!}$.

Ans. (a) *Both A and R are true and R is the correct explanation of A.*

Explanation: We know that,

$$^nP_r = P(n, r) = \frac{n!}{(n-r)!}$$

$$^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!}$$

$$= \frac{6\times5\times4\times3\times2!}{2!}$$

$$= 360$$

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

14. *A dentist conducts a team to take surveys of people in his locality about using toothpaste. A survey team has some persons and the survey team owner makes a team out of total persons available at that time. If he has a group of 9 persons available at that time out of which 5 are men and 4 are women.*

(A) In the commitee if it is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

(B) If $P(2n - 1, n) : P(2n + 1, n - 1) = 22:7$, find n.

(C) From a team of 6 students, in how many ways can we choose a captain and vice-captain assuming one person can not hold more than one position?

Ans (A) There are 9 seats, out of which 4 are at even places and rest are at odd places.

Thus, there ae 4 even places.

So, 4 women can be seated in 4 even places in 4! ways.

In rest of the places, five men can be placed in 5! ways.

Hence, required number of ways = 4! × 5!

= 24 × 120

= 2880

(B) We have,

$$^{2n-1}P_n : {}^{2n+1}P_{n-1} = 22:7$$

$$\frac{\frac{(2n-1)!}{(2n-1-n)!}}{\frac{(2n+1)}{(2n+1-n+1)!}} = \frac{27}{7}$$

$$\frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)!}{(2n+1)!} = \frac{22}{7}$$

$$\Rightarrow \frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)(n+1)n(n-1)!}{(2n+1)(2n)(2n-1)!} = \frac{22}{7}$$

$$\Rightarrow \frac{(n+2)(n+1)n}{(2n+1)(2n)} = \frac{22}{7}$$

$$\Rightarrow \frac{(n+2)(n+1)}{2(2n+1)} = \frac{22}{7}$$

$$7(n^2 + 3n + 2) = 44(2n + 1)$$

$$7n^2 + 21n + 14 = 88n + 44$$

$$7n^2 - 67n - 30 = 0$$

$$7n^2 - 70n + 3n - 30 = 0$$

$$7n(n - 10) + 3(n - 10) = 0$$

$$\Rightarrow (n - 10)(7n + 3) = 0$$

$$\Rightarrow n = 10, \text{ or } n = \frac{-3}{7}$$

$$\therefore n = 10$$

[$\because$ n can't be negative and fraction]

(C) From a team of 6 students, two students are to be chosen in such a way that one student will hold only one position.

Here, the no. of ways of choosing a captain and vice-captain is the permutation of 6 different things taken 2 at a time.

So, $^6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = 30$

15. *Sumit works at a book shop. While arranging some books on the book shelf, he observed that there are 5 History books, 3 Mathematics books and 4 Science books which are to be arranged on the shelf.*

(A) In how many ways can he select either a History or a maths book?

(a) 10 (b) 8

(c) 20 (d) 60

(B) If he selects 2 History books, 1 Math book and 1 Science book to arrange them, then find the number of ways in which selection can be made.

(a) 200 (b) 220

(c) 240 (d) 260

(C) Find the number of ways, if the books of the same subject are put together.

(a) 4! .2! .3! (b) 2! .3! .2! .5!

(c) 5! .2! .4! (d) 3! .5! .4!

(D) If we are given the number of arrangement of books are $^5P_2 \times {}^3P_1 \times {}^4P_1$, then the arrangement is in the manner:

(a) 2 History books, 2 Maths books, 3 Science books respectively.

(b) 2 History books, 3 Maths books, 2 Science books respectively.

(c) 3 History books, 2 Maths books, 2 Science books respectively.

(d) None of these

(E) In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together?

(a) 41472 (b) 42000

(c) 30000 (d) 50208

Ans. (A) *(b) 8*

Explanation: A History book can be selected in 5 ways and a Math book can be selected in 3 ways.

Required number of ways = 5 + 3 = 8

[Using addition principle]

(B) *(c) 240*

Explanation: Now, 2 History books can be chosen in $^5P_2$ ways, 1 Maths book can be chosen in $^3P_1$ ways and 1 Science book can be chosen in $^4P_1$ ways.

$\therefore$ Required number of ways = $^5P_2 \times {}^3P_1 \times {}^4P_1$

= 240

(C) *(d) 3! .5! .4!*

Explanation: Number of ways of arranging History books = 5!

Number of ways of arranging Maths books = 3!

Number of ways of arranging Science books = 4!

$\therefore$ Required number of ways if the books of same subject are put together = 3! .5! .4!

(D) *(d) None of these*

Explanation: The number of arrangements of books $^5P_2 \times {}^3P_1 \times {}^4P_1$ represents the arrangement of 2 History books, 1 Maths book and 1 Science book respectively.

(E) *(a) 41472*

Explanation: First we take books of a particular subject as one unit.

Thus, there are 4 units which can be arranged in 4! = 24 ways.

Now in each of arrangements.

Mathematics books can be arranged in 3! ways.

History books in 4! ways.

Chemistry books in 3! ways.

And biology books in 2! ways.

Thus, the total number of ways = 4! × 3! × 4! × 3! × 2!

= 24 × 6 × 24 × 6 × 2.

= 41472

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

16. **In an examination there are three multiple choice questions and each question has 4 choices. Find the number of ways in which a student can fail to get all answer correct.**

[Delhi Gov. QB 2022]

Ans. Since, each question has 4 options.

i.e., there are 4 choices or 4 ways to answer a question.

$\therefore$ Number of ways to answer 3 questions is

$$4 \times 4 \times 4 = 64$$

Out of 64 ways, there is only one way which has all the answer correct.

So, number of ways in which a student fails to get all answer correct is $64 - 1 = 63$ ways.

17. **If $^{16}P_{r-1} : {}^{15}P_{r-1} = 16 : 7$ then find r.**

Ans. Given, $^{16}P_{r-1} : {}^{15}P_{r-1} = 16 : 7$

We know that $^{n}P_r = \dfrac{n!}{(n-r)!}$

$$n! = n \times (n-1)!$$

Here,

$$^{16}P_{r-1} : {}^{15}P_{r-1} = 16 : 7$$

$$\frac{16!}{(16-r+1)!} \times \frac{(15-r+1)!}{15!} = \frac{16}{7}$$

$$\frac{16}{(17-r)!} \times (16-r)! = \frac{16}{7}$$

$$\frac{16}{17-r} = \frac{16}{7}$$

$$17 - r = 7$$

$$10 = r$$

Hence, $r = 10$

18. **Evaluate $2 \times 5! - 3 \times 4!$**

Ans. $2 \times 5! - 3 \times 4! = 2 \times 5 \times 4 \times 3 \times 2 \times 1 - 3 \times 4 \times 3 \times 2 \times 1$

$= 240 - 72$

$= 168$

19. **Find the number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9 where no digits are repeated.**

[Hint: Besides 4-digit integers greater than 7000, five digit integers are always greater than 7000]. **[NCERT Exemplar]**

Ans. According to the question,

Digits that can be used = 3, 5, 7, 8, 9

Since, no digits can be repeated.

The number of integers is $^5P_5 = 5!$

$= 120$

For a four-digit integer to be greater than 7000, the four-digit integer should begin with 7, 8 or 9.

The number of such integer

$= 3 \times {}^4P_3$

$= 3 \times 24$

$= 72$

Therefore, the total no. of ways

$= 120 + 72$

$= 192$

20. **A gentleman has 6 friends to invite. In how many ways can he send invitation cards to them if he has three servants to carry the cards?** **[Delhi Gov. QB 2022]**

Ans. Invitation cards may be sent to each of the six friends by anyone of the three servants in 3 ways.

$\therefore$ Required number of ways $= 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^6 = 729$

Hence, 729 ways are possible.

21. **Evaluate $\dfrac{6}{5}.5! - \dfrac{5}{4}.4!$.**

Ans. We have, $\dfrac{6}{5}.5! - \dfrac{5}{4}.4!$

$= \dfrac{6}{5} \times 5 \times 4 \times 3 \times 2 \times 1 - \dfrac{5}{4} \times 4 \times 3 \times 2 \times 1$

$= 144 - 30$

$= 114$

22. **Find the number of different four digit numbers that can be formed with digits 2, 3, 4, 7 and using each digit only once.**

[NCERT Exemplar]

Ans. We know that,

$$^{n}P_r = \frac{n!}{(n-r)!}$$

Four digit numbers that can be formed with digits 2, 3, 4, 7 and using each digit only once

Therefore, number of different four digit numbers

$= {}^4P_4$

$= 4!$

$= 24$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

23. In how many ways can the letters of the word "ABACUS" be arranged such that the vowels always appear together?

[Delhi Gov. QB 2022]

Ans. In the ABACUS, there are 3 vowels 2A's and U.

Number of letters in ABACUS are 6.

Let us take the vowels as one, so number of letters now = 4.

∴ Number of ways in which vowels occur together = 4!

But the 3 vowels can rearrange amongst themselves in $\frac{3!}{2!}$ ways as 'A' appears twice.

Hence, the total number of ways in which vowels occur together = $4! \times \frac{3!}{2!} = 72$.

24. If $^{12}P_{x+1} > 2\ ^{12}P_x$ then find the set of values of x.

Ans. We have $\quad {}^{12}P_{x+1} > 2\ {}^{12}P_x$

$$\frac{12!}{(11-x)!} > 2\frac{12!}{(12-x)(11-x)!}$$

$$1 > 2\frac{1}{(12-x)}$$

$$(12-x) > 2$$

$$x < 10$$

Hence, the set of values of x is {0, 1, 2 ... 9}.

25. Find r, if $^nP_r = 600$ and $^nC_r = 100$.

Ans. We know that, $^nP_r = {}^nC_r\ r!$.

Given that, $\quad ^nP_r = 600$ and $^nC_r = 100$

∴ $\quad 600 = 100 \times r!$

⇒ $\quad r! = 6$

⇒ $\quad r! = 3!$

⇒ $\quad r = 3$

26. Find the number of different words that can be formed from the letters of the word 'TRIANGLE' so that no vowels are together.

[NCERT Exemplar]

Ans. We know that,

$$^nP_r = \frac{n!}{(n-r)!}$$

According to the question,

Total number of vowels in 'TRIANGLE' = 3(A, E, I)

Five consonants can be arranged in 5! ways.

Now, we have six space between these five consonants to arrange vowels. So, the number of ways to arrange these vowels are $^6P_3$.

The vowels can be placed in

$$^6P_3 = \frac{6!}{3!}$$

$$= 120$$

The number of ways, in which consonants can be arranged = 5! = 120

Therefore, total number of ways = $5! \times {}^6P_3$

$$= 120 \times 120$$

$$= 14400$$

27. Find r, if $7\ ^6P_r = \frac{1}{3}\ ^7P_{r+2}$.

Ans. We have,

$$7\ ^6P_r = \frac{1}{3}\ ^7P_{r+2}$$

$$\Rightarrow \quad 7 \times \frac{6!}{(6-r)!} = \frac{1}{3} \times \frac{7!}{[7-(r+2)]!}$$

$$\Rightarrow \quad 7 \times \frac{6!}{(6-r)!} = \frac{1}{3} \times \frac{7!}{(5-r)!}$$

$$\Rightarrow \quad 7 \times \frac{6!}{(6-r)!} = \frac{1}{3} \times \frac{7 \times 6!}{(5-r)!}$$

$$\Rightarrow \quad \frac{1}{(6-r)(5-r)!} = \frac{1}{3} \times \frac{1}{(5-r)!}$$

$$\Rightarrow \quad \frac{1}{(6-r)} = \frac{1}{3}$$

$$\Rightarrow \quad (6-r) = 3$$

$$\Rightarrow \quad r = 3$$

28. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated. **[NCERT Exemplar]**

Ans. (i) Thousand's place can be filled with 6 alone.

Hence, the number of ways = 1

6			

Unit place can be filled with either 0 or 5.

Hence, number of ways = 2

6			0 or 5

Hundred's place can be filled with the remaining 8 digits.

Hence, number of ways = 8

6	9, 8, 7, 4, 3, 2, 1 (5 or 0)		(0 or 5)

Ten's place can be filled with 7 digits.
Number of ways = 7

6	9, 8, 7, 4, 3, 2, 1	Remaining 7	0 or 5

Thus, required number will be

$= 1 \times 8 \times 7 \times 2$

$= 112$

29. **Find the sum of all four digit numbers that can be formed by the digits 1, 3, 5, 7, 9 without repetition. [Diksha]**

Ans. Total no. of digits are 5, i.e.,

$n = 5$

Total no. of digits required is 4, i.e.,

$r = 4$

Therefore,

No. of 4 digits no. that can be formed $= {}^nP_r$

$= {}^5P_4$

$= \frac{5!}{1!} = 120$

No. of times each digit will appear $= \frac{120}{5} = 24$

Sum of digits at unit place

$= 24(1 + 3 + 5 + 7 + 9)$

$= 24 \times 25 = 600$

Sum of all numbers $= 600 \times 1000 + 600 \times 100 + 600 \times 10 + 600$

$= 666600$

Hence, the sum of all 4 digit numbers is 666600.

30. **How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once? [Diksha]**

Ans. Number of letters in word EQUATION = 8

$n = 8$

If all letters of the word used at a time

$r = 8$

Total numbers formed $= {}^nP_r$

$= {}^8P_8$

$= \frac{8!}{(8-8)!}$

$= \frac{8!}{0!}$

$= \frac{8!}{1}$

$= 8!$

$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 40320$

Thus, required number of words that can be formed = 8! = 40320.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

31. **How many different four letter words can be formed (with or without meaning) using the letters of the word "MEDITERRANEAN" such that the first letter is *E* and the last letter is *R*? [Delhi Gov. QB 2022]**

Ans. In the given word, there are 13 letters in total.

Since the two letters, E and R have fixed place. So, we don't have to select and arrange these letters.

So, now we only have to select two letters.

So, we have 11 letters left out of which 2E's, 2N's, 2A's.

So, the total number of ways to select the two different letters are 8 × 7 = 56.

The total number of ways to select the two same letters are 3.

So, the total number of required four-letter words is 56 + 3 = 59.

Therefore, the total number of four-letter words that can be formed from the given word such that the first letter is E and the last letter is R is 59.

Hence, option B is the correct answer.

32. **How many automobile licence plates can be made if each plate contains two different letters followed by three different digits? [NCERT Exemplar]**

Ans. According to the question,

Number of letters in automobile licence plates

= 2

We know that, there are 26 alphabets.

So, letters can be arranged without repetition in the following number of ways,

$= 26 \times 25$

$= 650$

Number of digits in automobile licence plates

= 3

We know that, there are 10 digits

0	1	2	3	4	5	6	7	8	9

Hence the number of digits without repetitions

$= 10 \times 9 \times 8$

$= 720$

Therefore, the total number of way automobile licence plates

$= 720 \times 650$

$= 468000$

33. Find the value of n such that $\frac{{}^nP_5}{{}^{n-1}P_5} = \frac{4}{3}$, $n > 4$.

Ans. We have,

$$\frac{{}^nP_5}{{}^{n-1}P_5} = \frac{4}{3}$$

$$\Rightarrow \quad 3\,{}^nP_5 = 4\,{}^{n-1}P_5$$

$$\Rightarrow \quad 3\times\frac{n!}{(n-5)!} = 4\times\frac{(n-1)!}{[(n-1)-5]!}$$

$$\Rightarrow \quad 3\times\frac{n!}{(n-5)!} = 4\times\frac{(n-1)!}{(n-6)!}$$

$$\Rightarrow \quad 3\times\frac{n(n-1)!}{(n-5)(n-6)!} = 4\times\frac{(n-1)!}{(n-6)!}$$

$$\Rightarrow \quad \frac{3n}{n-5} = 4$$

$$\Rightarrow \quad 3n = 4n - 20$$

$$\Rightarrow \quad n = 20$$

34. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then what is the rank of the word RACHIT?

[Hint: In each case number of words beginning with *A, C, H, I* is 5!] [NCERT Exemplar]

Ans. According to the question,

R	A	C	H	I	T

Arranging in alphabetical order, we get

ACHIRT

Number of words that can start with A = 5!

Number of words that can start with C = 5!

Number of words that can start with H = 5!

Number of words that can start with I = 5!

Total = 5! + 5! + 5! + 5!

= 120 + 120 + 120 + 120

= 480

Since, the first word that can start with R is RACHIT

Number of words that can start with R = 1

Hence, we obtain that,

The rank of the word RACHIT = 480 + 1 = 481

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

35. Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements. [NCERT Exemplar]

Ans. We know that,

$${}^nP_r = \frac{n!}{(n-r)!}$$

According to the question,

W_1 can occupy chairs marked 1 to 4 in 4 different ways.

Chair	1	2	3	4	5	6	7	8
People	W_1, W_2	W_1, W_2	W_1, W_2	W_1, W_2				

W_2 can occupy 3 chairs marked 1 to 4 in 3 different ways.

So, total no. of ways in which women can occupy the chairs,

$${}^4P_2 = \frac{4!}{(4-2)!}$$

$$= \frac{(4\times3\times2\times1)}{(2\times1)}$$

$${}^4P_2 = 12$$

Now, 6 chairs will be remaining.

Chair	1	2	3	4	5	6	7	8
People	W_1	W_2						

M_1 can occupy any of the 6 chairs in 6 different ways.

M_2 can occupy any of the remaining 5 chairs in 5 different ways.

M_3 can occupy any of the remaining 4 chairs in 4 different ways.

So, total no. of ways in which men can occupy the chairs.

$${}^6P_3 = \frac{6!}{(6-3)!}$$

$$= 120$$

Hence, total number of ways in which men and women can be seated

$${}^4P_2 \times {}^6P_3 = 12 \times 120$$

$$= 1440$$

COMBINATIONS AND ITS APPLICATIONS 2

TOPIC 1

COMBINATIONS

A combination is an unordered collection of some or all objects in a set.

The number of combination of r objects that can be selected from n different objects where $(0 < r \leq n)$ is given by

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Difference between Combination and Permutation

This will help you to understand clearly the difference between permutations and combinations.

Selection Combination	Arrangement Permutation
ABC	*ABC, ACB, BAC, BCA, CAB, CBA*
ABD	*ABD, ADB, BAD, BDA, DAB, DBA*
ACD	*ACD, ADC, CAD, CDA, DAC, DCA*
BCD	*BCD, BDC, CBD, CDB, DBC, DCB*
Total 4 combinations	24 permutations

Theorem 1

$$^nP_r = {}^nC_r \cdot r!, 0 < r \leq n$$

Proof: The number of combinations of n distinct objects taken r at a time is, ^nC_r. In these combinations, r things can be arranged among themselves in $r!$ ways. So, we have $r!$ permutations.

Therefore, the total number of permutations of n different things taken r at a time is $^nC_r \times r!$, which is equal to ^nP_r

$$\therefore \quad ^nP_r = r! \times {}^nC_r, 0 < r \leq n$$

Hence, proved.

Theorem 2

$$^nC_r = {}^nC_{n-r}, 0 \leq r \leq n$$

Important

This theorem is used to simplify the calculation when r is large.

Theorem 3

$$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

Proof:

$$\text{L.H.S.} = {}^nC_r + {}^nC_{r-1}$$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{n!(n-r+1)}{r!(n-r+1)(n-r)!} + \frac{n!r}{r(r-1)!(n-r+1)!}$$

$$= \frac{n!(n-r+1)}{r!(n-r+1)!} + \frac{n!r}{r!(n-r+1)!}$$

$$= n!\left[\frac{n-r+1+r}{r!(n-r+1)!}\right]$$

$$= \frac{(n+1)n!}{r!(n+1-r)!}$$

$$= \frac{(n+1)!}{r!(n+1-r)!}$$

$$= {}^{n+1}C_r = \text{R.H.S}$$

Hence, proved.

Theorem 4

If $^nC_x = {}^nC_y$, then either $x = y$ or $x + y = n$.

Example 2.1: If $^nC_8 = {}^nC_6$, find $^nC_2$.

Ans. If $^nC_x = {}^nC_y$ then either $x = y$ or $x + y = n$.

$$\therefore \quad ^nC_8 = {}^nC_6$$

$$\Rightarrow \quad n = (8 + 6) = 14$$

Now, $^nC_2 = {}^{14}C_2$

$$= \frac{14}{2} \times \frac{13}{1} \times {}^{12}C_0$$

$$\left[\because \ ^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}\right]$$

$$= \frac{14}{2} \times \frac{13}{1} \times 1 = 91$$

Example 2.2: How many words with or without meaning, each 2 of vowels and 3 consonants, can be formed from the letters of the word DAUGHTER? [NCERT]

Ans. There are 3 vowels and 5 consonants in the word DAUGHTER out of which 2 vowels and 3 consonants can be chosen in $^3C_2 \times {}^5C_3$ ways. These selected five letters can now be arranged in 5! ways.

Hence, the required number of words

$$= {}^3C_2 \times {}^5C_3 \times 5! = 3 \times 10 \times 120 = 3600$$

Example 2.3: The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabets? [NCERT]

Ans. Out of 5 vowels and 21 consonants, 2 vowels and 2 consonants can be chosen in

$$^5C_2 \times {}^{21}C_2 \times 4! = 10 \times 210 \times 24 = 50400.$$

Example 2.4: In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together? [NCERT]

Ans. Since, boys are to be separated. Therefore, let us first seat 5 girls. This can be done in 5! ways. For each such arrangement, three boys can be seated only at the cross-marked places.

$$\times G \times G \times G \times G \times G \times$$

There are 6 crossed marked places and three boys can be seated in $^6C_3 \times 3!$ ways. Hence, by the fundamental principle of counting, the total number of ways is $5! \times {}^6C_3 \times 3! = 14400$.

Example 2.5: If the ratio $^{2n}C_3 : {}^nC_3$ is equal to 11 : 1, find *n*. [NCERT]

Ans. We have,

$$^{2n}C_3 : {}^nC_3 = 11 : 1$$

$$\Rightarrow \quad \frac{^{2n}C_3}{^nC_3} = \frac{11}{1}$$

$$\Rightarrow \quad \frac{\frac{(2n)!}{(2n-3)!3!}}{\frac{n!}{(n-3)!(3!)}} = \frac{11}{1}$$

$$\Rightarrow \quad \frac{(2n)!}{(2n-3)!} \times \frac{(n-3)!}{n!} = \frac{11}{1}$$

$$\Rightarrow \quad \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = \frac{11}{1}$$

$$\Rightarrow \quad \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$$

$$\frac{4(2n-1)}{n-2} = \frac{11}{1}$$

$$\Rightarrow \quad 8n - 4 = 11n - 22$$

$$\Rightarrow \quad 3n = 18$$

$$\Rightarrow \quad n = 6$$

Example 2.6: Case Based:

In the BCCI board, Jagmohan Dalmia is a Cricket team selector. He selected a cricket team from 17 players in which only 5 players can bowl. Then, answer the following questions which are based on it.

(A) Find the number of ways in which exactly 4 bowlers must be included out of 11 players.

(B) Assertion (A): The concept used for finding the required number of players is combination.

Reason (R): The combination is the unordered collection of some or all objects in a set.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

(C) In how many ways exactly 3 bowlers must include out of 11 players.

(a) 3950 (b) 4950

(c) 5950 (d) 6950

(D) If the number of ways of selecting exactly 3 bowlers must include out of 11 players is *abcd* then the value of (*a* + *b* + *c* + *d*) is:

(a) 10 (b) 18

(c) 20 (d) 25

(E) Find the number of ways of selecting exactly 4 bowlers must include out of 11 players is *abcd* then the value of (*a* + *b* + *c* + *d*).

Ans. (A) Given that, total number of players is 17.

We have to select 11 players including exactly 4 bowlers.

Hence, 4 bowlers will be selected from 5 bowlers and remaining 7 players will be selected from 12 players.

Now, 4 bowlers out of 5 bowlers can be selected in $^5C_4$ ways.

7 players out of 12 players can be selected in $^{12}C_7$ ways.

∴ Total number of ways selecting 11 players

$$= {}^5C_4 \times {}^{12}C_7$$

$$= {}^5C_1 \times {}^{12}C_5 \qquad [\because {}^nC_r = {}^nC_{n-r}]$$

$$= 5 \times \frac{12!}{5!7!}$$

$$= 5 \times \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1}$$

$= 5 \times 11 \times 9 \times 8 = 55 \times 72$
$= 3960$

Hence, he can select the team of 11 players in 3960 ways.

(B) *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: To find the number of required players the concept of combination is used.

(C) *(b) 4950*

Explanation: Given that, total number of players is 17.

We have to select 11 player including exactly 3 bowlers.

Hence, 3 bowlers will be selected from 5 bowlers and remaining 8 players will be selected from 12 players.

Now, 3 bowlers out of 5 bowlers can be selected in ${}^5C_3$ ways.

8 players out of 12 players can be selected in ${}^{12}C_8$ ways.

∴ Total number of ways selecting 11 players

$= {}^5C_3 \times {}^{12}C_8$

$$= \frac{5!}{2!3!} \times \frac{12!}{8!4!}$$

$$= \frac{5 \times 4}{2} \times \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2}$$

$= 10 \times 11 \times 5 \times 9$

$= 4950$

(D) *(b) 18*

Explanation: Here, $a = 4, b = 9, c = 5, d = 0$ [From (C)]

$\therefore a + b + c + d = 4 + 9 + 5 + 0 = 18$

(E) Here, $a = 3, b = 9, c = 6$ and $d = 0$ [from (A)]

$\therefore a + b + c + d = 3 + 9 + 6 + 0 = 18.$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Evaluate ${}^{22}C_2$.

(a) 231 (b) 232
(c) 233 (d) 234

Ans. *(a) 231*

Explanation: We have,

$${}^{22}C_2 = \frac{22!}{20!2!} = \frac{22 \times 21 \times 20!}{20! \times 2 \times 1}$$

$$= \frac{22 \times 21}{2} = 21 \times 11$$

$= 231$

2. If ${}^{12}C_x < 2\ {}^{12}C_{x+1}$ then set of values of x is:

(a) [1, 10] (b) (7, ∞)
(c) {1, 2, 3} (d) [1, 8]

Ans. *(d) [1, 8]*

Explanation: We have,

$${}^{12}C_x < 2\ {}^{12}C_{x+1}$$

$$\frac{12!}{x!(12-x)!} < 2 \frac{12!}{(11-x)!(x+1)!}$$

$$\frac{1}{12-x} < 2 \frac{1}{x+1}$$

$x + 1 < 24 - 2x$

$x + 2x < 24 - 1$

$3x < 23$

$$x < \frac{23}{3}$$

$x = 7.65\ \_\ \_\ \_$

3. **A committee has 5 men and 6 women. What are the number of ways of selecting 2 men and 3 women from the given committee?**

(a) 100 (b) 204
(c) 200 (d) 300 **[Diksha]**

Ans. *(c) 200*

Explanation: The number of ways to select two men and three women $= {}^5C_2 \times {}^6C_3$

No. of ways of selecting 2 men and 3 women from committee

$$= \frac{(5 \times 4)}{(2 \times 1)} \times \frac{(6 \times 5 \times 4)}{(3 \times 2)}$$

$= 200$

4. If ${}^{2x+3}C_{2x} - {}^{2x+2}C_{2x-1} = 15(2x + 1)$, then x is:

(a) 13 (b) 14
(c) 27 (d) 15

Ans. *(b) 14*

Explanation: We have,

$$^{2x+3}C_{2x} - {}^{2x+2}C_{2x-1} = 15\,(2x+1)$$

$$\Rightarrow \quad \frac{(2x+3)!}{2x!3!} - \frac{(2x+2)!}{(2x-1)!3!} = 15(2x+1)$$

$$\Rightarrow \quad \frac{(2x+2)(2x+1)}{2} = 15(2x+1)$$

$$\Rightarrow \quad x+1 = 15$$

$$\Rightarrow \quad x = 14$$

5. If $^nC_{12} = {}^nC_8$, then n is:

(a) 20 (b) 12

(c) 6 (d) 30

[NCERT Exemplar]

Ans. *(a) 20*

Explanation: According to the question,

$$^nC_{12} = {}^nC_8$$

We know that,

$$^nC_a = {}^nC_b$$

$$n = a + b$$

$$n = 12 + 8$$

$$n = 20$$

6. If $^nC_{10} = {}^nC_9$, then the value of $^nC_{19}$ is:

(a) 1 (b) 2

(c) 0 (d) 3

Ans. *(a) 1*

Explanation: Here,

$$^nC_p = {}^nC_k$$

$$n = p + k \text{ or } p = k$$

$$^nC_{10} = {}^nC_9$$

$$\Rightarrow \quad n = 10 + 9 = 19$$

Then $^{19}C_{19} = 1$

7. The number of diagonals of a polygon of 17 sides is:

(a) 225 (b) 350

(c) 119 (d) 210

Ans. *(c) 119*

Explanation: Here required number is

$^{17}C_2 - 17$ number of sides

number of ways of selecting 2 points,

$$\frac{17\times 16}{2} - 17$$

$\Rightarrow$ $17 \times 8 - 17$

$\Rightarrow$ 17×7

$\Rightarrow$ 119

8. $^nP_r = 840$, $^nC_r = 35$, then r is:

(a) 2 (b) 4

(c) 3 (d) 5 [Diksha]

Ans. *(b) 4*

Explanation:

$$\therefore \quad {}^nP_r = r!\,{}^nC_r$$

$$840 = r!\,35$$

$$r! = \frac{840}{35} = 24$$

$$r! = 4!$$

$$r = 4$$

9. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is:

(a) 60 (b) 120

(c) 7200 (d) 720

[NCERT Exemplar]

Ans. *(c) 7200*

Explanation: We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Total number of given vowels = 4

Total number of given consonant = 5

Total number of words that can be formed by 2 vowels and 3 consonants = $^4C_2 \cdot {}^5C_3$

$$= \frac{4!}{2!2!} \times \frac{5!}{3!2!}$$

$$\text{Total number of words} = 5! \times \frac{4!}{2!2!} \times \frac{5!}{3!2!}$$

$$= 7200$$

10. If $^{x-1}C_3 + {}^{x-1}C_4 > {}^xC_3$, then x is:

(a) $x > 5$ (b) $x > 6$

(c) $x > 7$ (d) none

Ans. *(c) $x > 7$*

Explanation: We have,

$$^{x-1}C_3 + {}^{x-1}C_4 > {}^xC_3$$

$$^xC_4 > {}^xC_3 \quad [\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$$

$$\Rightarrow \quad \frac{x!}{(x-4)!4!} > \frac{x!}{(x-3)!3!}$$

$$\frac{1}{4} > \frac{1}{x-3}$$

$$x - 3 > 4$$

$$x > 7$$

11. Everybody in a room shakes hands with everybody else. The total number of handshakes is 66. The total number of persons in the room is:

(a) 11 (b) 12

(c) 13 (d) 14

[NCERT Exemplar]

Ans. *(b)* 12

Explanation: We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Let the total number of handshakes = $^nC_2 = 66$

There should be two hands involved for a handshake.

$$\therefore \quad ^nC_2 = 66$$

$$\frac{n!}{2!(n-2)!} = 66$$

$$\Rightarrow \quad \frac{n(n-1)}{2} = 66$$

$$\Rightarrow \quad n^2 - n = 132$$

$$\Rightarrow (n-12)(n+11) = 0$$

$$n = 12 \text{ or } n = -11$$

Therefore, $n = 12$

12. There are 30 students in a group. If all shake hands with one another, how many handshakes are possible? [Diksha]

(a) 436 (b) 435

(c) 345 (d) 534

Ans. *(b)* 435

Explanation: Given,

Number of students = 30

A handshake needs 2 people. So total ways of two people shaking hands with each other $= {}^{30}C_2$

We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

where, $n = 30$ and $r = 2$

Therefore,

$$^{30}C_2 = \frac{30!}{2!(30-2)!}$$

$$= \frac{30 \times 29 \times 28!}{2 \times 1 \times 28!}$$

$$= \frac{30 \times 29}{2}$$

$$= 435$$

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

13. Assertion (A): Number of lines formed by joining n points on a circle $(n \geq 2)$ is $\frac{n(n-1)}{2}$.

Reason (R): $C(n, 3) = \frac{n(n-1)}{2}$.

Ans. *(c) (A) is true but (R) is false.*

Explanation: Number of lines is $^nC_2 = \frac{n(n-1)}{2}$

$$C(n, 3) = \frac{n!}{3!(n-3)!} = \frac{n(n-1)(n-2)}{6}$$

14. Assertion (A): The product of five consecutive natural numbers is divisible by 4!

Reason (R): Product of n consecutive natural numbers is divisible by $(n+1)!$.

Ans. *(c) (A) is true but (R) is false.*

Explanation: Product of n consecutive natural numbers

$$= (m+1)(m+2)(m+3) \ldots (m+n),\ m \in W$$

$$= \frac{(m+n)!}{m!} = n! \times \frac{(m+n)!}{m!\,n!} = n! \times {}^{m+n}C_m$$

Product is divisible by $n!$ and so it is always divisible by $(n-1)!$ but not by $(n+1)!$

15. Assertion (A): Number of rectangles on a chess board is $^8C_2 \times {}^8C_2$.

Reason (R): To form a rectangle, we have to select any two of the horizontal line and any two of the vertical line.

Ans. *(d) (A) is false but (R) is true.*

Explanation: To form a rectangle, we have to select any two of the horizontal line and any two of the vertical line.

In a chess board, there are 9 horizontal and 9 vertical lines. Number of rectangles of any size are $^9C_2 \times {}^9C_2$.

16. Assertion (A): If n is a positive integer, then $n(n^2-1)(n+2)$ is divisible by 24.

Reason (R): Product of r consecutive positive integers is divisible by $r!$.

Ans. *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: $n(n^2-1)(n+2) = (n-1)n(n+1)(n+2)$ is the product of four consecutive positive integers and hence it is divisible by 24.

17. Assertion (A): The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Reason (R): The number of ways of choosing any 3 places, from 9 different places is ${}^9C_3$.

Ans. *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: Let the number of ways of distributing *n* identical objects among *r* persons such that each person gets at least one object is same as the number of ways of selecting (*r* – 1) places out of (*n* – 1) different places, i.e., ${}^{n-1}C_{r-1}$.

$\therefore$ ${}^{10-1}C_{4-1}$

The number of ways will become ${}^9C_3$.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

18. *Riya and her 5 friends went for a trip to Shimla. They stayed in a hotel. There were 4 vacant rooms A, B, C and D. Out of these 4 vacant rooms, two rooms A and B were double share rooms and two rooms C and D can contain one person each.*

(A) The number of ways in which in which room A can be filled is:

(a) 10 (b) 15

(c) 20 (d) 25

(B) If room *A* and *B* are already filled each, then the number of ways in which room C and be filled is:

(a) 2 (b) 4

(c) 6 (d) 8

(C) The total number of ways of accommodating Riya and her friends in these 4 vacant rooms is:

(a) 150 (b) 160

(c) 170 (d) 180

(D) If room A is filled with 2 persons, then the number in which rooms C and D can be filled is:

(a) 4 (b) 12

(c) 8 (d) 10

(E) The number of ways in which 10 digit numbers can be written using the digits 1 and 2 is:

(a) 2^{10} (b) ${}^{10}C_2$

(c) 10! (d) ${}^{10}C_1 + {}^9C_2$

Ans. (A) *(b) 15*

Explanation: Total members = 6

$\because$ Room *A* is a double shared room.

$\therefore$ The number of ways in which room *A* can be filled = ${}^6C_2 = 15$

(B) *(a) 2*

Explanation: Now, rooms *A* and *B* can be filled with 2 members each and room *C* can be filled with 1 person.

$\therefore$ Required number of ways = ${}^2C_1 = 2$

(C) *(d) 180*

Explanation: Required number of ways

$= 15 \times 6 \times 2 \times 1 = 180$

(D) *(b) 12*

Explanation: As, room A is filled with 2 persons

Now, the remaining persons = 4

Given that room C and D can occupy 1 person each.

$\therefore$ The number of ways in which rooms C and D can be filled = ${}^4C_1 \times {}^3C_1 = 12$

(E) *(a) 2^{10}*

Explanation: Given digits are 1 and 2.

Here, each place can be filled in two ways either with 1 or 2 and every place has two chances.

Therefore, the number of ways 10 digit numbers can be written using the digits 1 and 2 is 2^{10}.

19. *Two friends Swati and Komal are playing cards. Swati asks Komal to choose any four cards from a pack of 52 cards. Now, based on this answer the following:*

(A) In how many ways can Komal select 4 cards from same suit and she select all 4 cards from different suits?

(B) In how many ways can Komal select 4 cards of different suit?

(C) In how many ways can she select 4 face cards from all face cards?

Ans. (A) Komal can select 4 cards from same suit either 4 hearts or 4 diamonds or 4 spades or 4 clubs.

i.e., $^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 = 4 \times {}^{13}C_4$

(B) She can select 4 cards from different suits as 1 heart, 1 diamond, 1 spade and 1 club

i.e., $^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$= 13 \times 13 \times 13 \times 13 = (13)^4$

(C) In a pack of 52 cards, there are 12 face cards and 40 non-face cards.

$\therefore$ Number of ways of selecting 4 face cards from 12 face cards

$$^{12}C_4 = \frac{12!}{4!8!} = \frac{12\times11\times10\times9\times8!}{4\times3\times2\times1\times8!} = 495$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

20. **Find r, if $^nP_r = 2880$ and $^nC_r = 120$.**

Ans. We know that $^nP_r = {}^nC_r\, r!$.

Given that $^nP_r = 2880$ and $^nC_r = 120$

$\therefore \quad 2880 = 120 \times r!$

$\Rightarrow \quad r! = 24$

$\Rightarrow \quad r! = 4!$

$\Rightarrow \quad r = 4$

21. **How many committees of five persons with a chairperson can be selected from 12 persons? [Hint: Chairman can be selected in 12 ways and remaining in $^{11}C_4$]. [NCERT Exemplar]**

Ans. We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Number of ways a chairperson can be selected $= 12$

Selection of 4 other people

$$= {}^{11}C_4 = \frac{11!}{4!7!} = 330$$

Selection of 5 people

$= 330 \times 12 = 3960$

22. **Find r, if $^{10}C_{2r} = {}^{10}C_{r+2}$.**

Ans. We know that if $^nC_a = {}^nC_b$, then either

$a = b$ or $n = a + b$

Given that, $^{10}C_{2r} = {}^{10}C_{r+2}$

Then, either $2r = r + 2$ or $10 = 2r + (r + 2)$.

i.e., either $r = 2$ or $r = \frac{8}{3}$.

But, $r = \frac{8}{3}$ is not possible, as r is always a whole number.

Hence, $r = 2$.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

23. **In how many ways can a student choose a programme of 7 courses if 9 courses 5 optional and 4 compulsory) are available and 4 specific courses are compulsory for every student?**

Ans. To choose a programme of 7 courses (3 optional and 4 compulsory) from 9 courses (including 5 optional and 4 compulsory).

Here, the order is not important.

So, each selection is a combination.

Number of ways of selecting 3 optional from 5

$$= {}^5C_3 = \frac{5!}{3!2!}$$

$= 10$

Number of ways of selecting 4 from 4 compulsory courses = $^4C_4$

= 1

Hence, required number of ways = 10 × 1

= 10

24. **How many words, with or without meaning each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE?** **[Delhi Gov. Term-2 SQP 2022]**

Ans. INVOLUTE contains 4 vowels (I, O, U, E) and 4 consonants (N, V, L, T).

3 vowels can be selected out of 4 vowels by $^4C_3$ ways and 2 consonants can be selected out of 4 consonants by $^4C_2$ ways.

∴ Number of words formed using 3 vowels and 3 consonants

$= {^4C_3} \times {^4C_2} \times 5! = 4 \times 6 \times 120 = 2880$

Hence total number of words is 2880 which contains 3 vowels and 2 consonants.

25. **A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.** **[NCERT Exemplar]**

Ans. We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

No of questions in group $A = 6$

No of questions in group $B = 6$

According to the question,

The different ways in which the question can be attempted are,

Group A	2	3	4	5
Group B	5	4	3	2

Hence, the number of different ways of doing questions

$= ({^6C_2} \times {^6C_5}) + ({^6C_3} \times {^6C_4}) + ({^6C_4} \times {^6C_3}) + ({^6C_5} \times {^6C_2})$

$= (15 \times 6) + (20 \times 15) + (15 \times 20) + (6 \times 15)$

$= 780$

26. **Out of 18 points in a plane, no three points are in the same line except five points which are collinear. Find the number of lines that can be formed joining the point.**

Ans. We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

According to the question,

Number of points = 18

Number of collinear points = 5

Number of lines formed by 18 points = $^{18}C_2$

Number of lines formed by 5 collinear points = $^5C_2$

The number of lines that can be formed joining the point, = $^{18}C_2 - {^5C_2} + 1$

$$= \frac{18!}{2!16!} - \frac{5!}{2!3!} + 1$$

$= 153 - 10 + 1$

$= 144$

∴ The total no. of ways of different lines formed are 144.

27. **There are two identical red, two identical black, and two identical white balls. In how many different ways can the balls be placed in the cells (Each cell to contain one ball) shown above such that balls of the same color do not occupy any two consecutive cells?** **[Diksha]**

Ans. **Case I:** 2 balls of the same colour and two balls of a different colour are arranged.

Two balls of the same colour and two balls of different colours can be arranged together in which two balls of the same colour are adjacent

$$= \frac{4!}{(2! \times 2!)} = 6 \text{ ways}$$

Therefore, total number of arrangements

$= 6 \times 3 = 18$ ways

Case II: Two colours out of 3 can be selected in

$= {^3C_1} = 3$ ways

Now 2 balls of each colour can be arranged alternatively in 2 ways

Thus 4 balls can be arranged (two of each colours) $= 3 \times 2 = 6$ ways

Hence, total number of arrangements

$= 18 + 6 = 24$ ways

28. **Determine n, if $^{2n}C_3 : {^nC_3} = 12 : 1$.**

Ans. We have, $^{2n}C_3 : {^nC_3} = 12 : 1$

$\Rightarrow \quad \frac{^{2n}C_3}{^nC_3} = \frac{12}{1}$

$\Rightarrow \quad {^{2n}C_3} = 12\, {^nC_3}$

$\Rightarrow \quad \frac{(2n)!}{3!(2n-3)!} = 12 \times \frac{n!}{3!(n-3)!}$

$\Rightarrow \quad \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3!(2n-3)!} = \frac{12 \times n(n-1)(n-2)(n-3)!}{3!(n-3)!}$

$\Rightarrow \quad 2(2n-1) = 6(n-2)$

$\Rightarrow \quad 4n - 2 = 6n - 12$

$\Rightarrow \quad n = 5$

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

29. **A boy has 4 movie tickets and 9 movie of his interest in the theater. Of these 9, he does not want to see Marvels Part II, unless Marvels Part I is also seen. In how many ways can he choose the 3 movies to be seen?**

Ans. To choose 4 movies from 9 movies. Marvels Part II is not to be seen, unless Marvels Part I is also seen.

Here, the order is not important.

So, each selection is a combination.

Case I: Marvels Part II is seen.

In this case, Marvels Part I is also seen.

Number of ways of selecting 2 movies from

$$7 \text{ movies} = {}^7C_2 = \frac{7!}{2!5!} = 21$$

Case II: Marvels Part II is not seen.

In this case, Marvels Part I may or may not be seen.

Number of ways of selecting 4 movies from

$$8 \text{ movies} = {}^8C_4 = \frac{8!}{4!4!} = 70$$

Hence, the required number of ways

$$= 21 + 70 = 91$$

30. **A committee of 7 has to be formed out of 9 boys and 4 girls. In how many ways can this be done when the committee consists of**

(A) exactly 3 girls

(B) atmost 3 girls?

[Delhi Gov. Term-2 SQP 2022]

Ans. A committee of 7 has to be formed from 9 boys and 4 girls.

(A) exactly 3 girls $= {}^9C_4 \times {}^4C_3$

$$= \frac{9!}{4!5!} \times \frac{4!}{3!1!}$$

$$= \frac{9\times8\times7\times6\times5!}{3\times2\times5!}$$

$$= 72 \times 7 = 504$$

(B) There are four possibilities,

(1) No girl and 7 boys

(2) 1 girl and 6 boys

(3) 2 girls and 5 boys

(4) 3 girls and 4 boys

$\therefore$ Committee consisting of atmost 3 girls

$$= {}^4C_0 \times {}^9C_7 + {}^4C_1 \times {}^9C_6 + {}^4C_2 \times {}^9C_5 + {}^4C_3 \times {}^9C_4$$

$$= 1 \times 36 + 4 \times 84 + 6 \times 126 + 126 \times 4$$

$$= 36 + 336 + 756 + 504$$

$$= 1632$$

31. **We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can selections be made?** **[NCERT Exemplar]**

Ans. We know that,

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

According to the question,

Case I: If both A and B are selected

$$= 1 \times 1 \times {}^6C_4 = \frac{6!}{4!(6-4)!} = \frac{6!}{4!2!} = 1$$

Case II: If neither A nor B are selected $= {}^6C_6 = 1$

If B is selected, but A is not selected $= 1 \times {}^6C_5$

$$= \frac{6!}{5!(6-5)!} = 6$$

Adding the result of both A and B being selected, neither A nor B being selected and B being selected but A not being selected.

Hence, total no. of ways are

$$= 15 + 1 + 6 = 22$$

32. **In an election, these are ten candidates and four are to be elected. A voter may vote for any number of candidates, not greater than the number to be elected. If a voter for at least one candidate, then find the number of ways in which he can vote.**

[Delhi Gov. GB 2022]

Ans. Total candidates = 10

Candidates to be selected = 4

Number of ways in which a voter may vote for atleast one candidates

$$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$$

$$= \frac{10!}{1!9!} + \frac{10!}{2!8!} + \frac{10!}{3!7!} + \frac{10!}{4!6!}$$

$$= 10 + 45 + 120 + 210$$

$$= 385$$

then the number of ways in which he can vote, is 385.

33. **Prove that:** $\dfrac{^nC_r}{^{n-1}C_{r-1}} = \dfrac{n}{r}$.

Ans. L.H.S. $= \dfrac{^nC_r}{^{n-1}C_{r-1}}$

$$= \frac{\left[\frac{n!}{r!(n-r)!}\right]}{\left[\frac{(n-1)!}{(r-1)![(n-1)-(r-1)]!}\right]}$$

$$= \frac{\left[\frac{n!}{r!(n-r)!}\right]}{\left[\frac{(n-1)!}{(r-1)!(n-r)!}\right]}$$

$$= \frac{n!}{r!(n-r)!} \times \frac{(r-1)!(n-r)!}{(n-1)!}$$

$$= \frac{n(n-1)!}{r(r-1)!(n-r)!} \times \frac{(r-1)!(n-r)!}{(n-1)!}$$

$$= \frac{n}{r}$$

= R.H.S.

34. **A convex polygon has 44 diagonals. Find the number of its sides. [Hint: Polygon of n sides has ($^nC_2 - n$) number of diagonals].**

[NCERT Exemplar]

Ans. We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Let the number of sides the given polygon have $= n$

Now,

The number of line segments obtained by joining n vertices $= {^nC_2}$

So, number of diagonals of the polygon

$$= {^nC_2} - n$$
$$= 44$$

$$\frac{n!}{2!(n-2)!} - n = 44$$

$$\frac{n(n-1)(n-2)!}{2(n-2)!} - n = 44$$

$$\frac{n(n-1)}{2} - n = 44$$

$$n^2 - 3n - 88 = 0$$
$$(n-11)(n+8) = 0$$
$$n = 11 \text{ or } n = -8$$

$\therefore$ The polygon has 11 sides.

35. **In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?**

[Delhi Gov. QB 2022]

Ans. Since there is no condition for positive (+) sign, fix them in a row.

+++++++

There are 6 places in between each plus and one before and one after these positive (+) sign.

i.e., these are 8 places for negative (–) sign and 5(–) negative signs are there.

$\therefore$ These negative (–) signs can be placed in $^8C_5$ ways.

$$^8C_5 = \frac{8!}{3!\,5!}$$

$$= \frac{8\times7\times6}{3\times2\times1} = 56$$

36. **A student has to answer 10 questions, choosing at least 4 from each of part A and B. If there are 6 questions in part A and 7 in part B. In how many ways can the student choose 10 questions?**

Ans. Total numbers of questions = 10

Questions in part $A = 6$

Questions in part $B = 7$

Selecting questions with at least 4 from each part A and part B, can from done in following way.

$$^6C_4 \times {^7C_6} + {^6C_5} \times {^7C_5} + {^6C_6} \times {^7C_4}$$

$$= \left(\frac{6!}{4!2!} \times \frac{7!}{6!1!}\right) + \left(\frac{6!}{5!1!} \times \frac{7!}{5!2!}\right) + \left(1 \times \frac{7!}{4!3!}\right)$$

$$\left[\because {^nC_r} = \frac{n!}{r!(n-r)!}\right]$$

$$= \left(\frac{6\times5\times7}{2}\right) + \left(\frac{6\times7\times6}{2}\right) + \left(\frac{7\times6\times5}{3\times2}\right)$$

= 105 + 126 + 35

= 266 ways

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

37. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these

(A) four cards are of same suit color?

(B) four cards belong to four different suits?

(C) four cards are face cards?

(D) two are red cards and two are black cards?

(E) cards are of the same colour?

Ans. Four cards can be chosen from 52 playing cards in ${}^{52}C_4$ ways.

Now, ${}^{52}C_4 = \frac{52!}{48!\,4!} = \frac{49\times50\times51\times52}{2\times3\times4} = 270725$

Hence, required number of ways = 270725

(A) There are four suits (diamond, spade, club and heart) of 13 cards each. Therefore, there are ${}^{13}C_4$ ways of choosing 4 diamond cards, ${}^{13}C_4$ ways of choosing 4 club cards, ${}^{13}C_4$ ways of choosing 4 spade cards and ${}^{13}C_4$ ways of choosing heart cards.

∴ Required number of ways

$= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$= 4 \times \frac{13!}{9!4!}$

$= 2860$

(B) There are 13 cards in each suit. Four cards drawn belong to four different units means one card is drawn from each suit. Out of 13 diamond cards one card can be drawn in ${}^{13}C_1$ ways.

Similarly, there are ${}^{13}C_1$ ways of choosing one club card, ${}^{13}C_1$ ways of choosing one spade card and ${}^{13}C_1$ ways of choosing one heart card.

∴ Number of ways of selecting one card from each suit $= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$= 4 \times {}^{13}C_1$

$= 4 \times \frac{13!}{1!12!}$

$= 4 \times 13$

$= 52$

(C) There are 12 face cards out of which 4 cards can be chosen in ${}^{12}C_4$ ways.

∴ Required number of ways

$= {}^{12}C_4$

$= \frac{12!}{4!8!}$

$= 495$

(D) There are 26 red cards and 26 black cards. Therefore, 2 red cards can be chosen in ${}^{26}C_2$ ways and 2 black cards can be chosen in ${}^{26}C_2$ ways. Hence, 2 red and 2 black cards can be chosen in ${}^{26}C_2 \times {}^{26}C_2$

$= \left(\frac{26!}{24!2!}\right)^2$

$= (325)^2$

$= 105625$ ways.

(E) Out of 26 red cards, 4 red cards can be chosen in ${}^{26}C_4$ ways. Similarly, 4 black cards can be chosen in ${}^{26}C_4$ ways.

Hence, 4 red or 4 black cards can be chosen in ${}^{26}C_4 + {}^{26}C_4$ ways

$= 2 \times \frac{26!}{4!22!} = 29900$ ways.

38. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find ${}^rC_2$.

[Hint: From equation using $\frac{{}^nC_r}{{}^nC_{r+1}}$ and $\frac{{}^nC_r}{{}^nC_{r-1}}$ to find the value of r]. **[NCERT Exemplar]**

Ans. We know that,

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

According to the question,

$${}^nC_{r-1} = 36,$$
$${}^nC_r = 84,$$
$${}^nC_{r+1} = 126$$
$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{84}{126}$$

$$\Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{84}{126} = \frac{2}{3}$$

$$2n - 2r = 3r + 3$$

$$\Rightarrow \quad 2n - 3 = 5r \quad \text{...(i)}$$

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{84}{36}$$

$$\Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = \frac{7}{3}$$

$$\Rightarrow \quad 3n - 3r + 3 = 7r$$

$$3n + 3 = 10r \quad \text{...(ii)}$$

From (i) and (ii), we get

$$2(2n - 3) = 3n + 3$$
$$4n - 3n - 6 - 3 = 0$$
$$n = 9$$

And $r = 3$

Now $${}^{r}C_2 = {}^{3}C_2 = \frac{3!}{2!}$$
$$= 3$$

39. In how many ways can 8 men and 7 women be seated along a round table if no two women sit together?

Ans. To arrange 8 men and 7 women, taken all at a time, along a round table such that no two women sit together.

Here, each arrangement is a stationary circular permutation.

We arrange 8 men along a circle. So, we arrange 8 men, taken all at a time.

So, number of ways in which men are arranged

$$= \frac{{}^{8}P_8}{8}$$
$$= \frac{8!}{8 \times 0!}$$
$$= 5040$$

We arrange 7 women so that the women occupy the empty places.

We first choose 7 places and then arrange 7 women in them.

(i) We choose 7 places from 8 places, for the women to sit.

Here, the order is not important.

So, each selection is a combination.

Number of ways of selecting 7 places from 8 places $= {}^{8}C_7 = \frac{8!}{7!1!} = 8$

So, number of such combinations = 8

(ii) We arrange women on the places in each of the above combinations.

Here, the order is important and the repetition of women cannot be done.

So, each arrangement is a permutation.

We arrange 7 women, taken all at a time.

Number of arrangements for each combination $= {}^{7}P_7 = \frac{7!}{0!} = 5040$

So, number of ways in which women are arranged $= 8 \times 5040 = 40320$.